



University of the Pacific
Scholarly Commons

Euler Archive - All Works

Euler Archive

1750

Methodus facilis computandi angulorum sinus ac tangentes tam naturales quam artificiales

Leonhard Euler

Follow this and additional works at: <https://scholarlycommons.pacific.edu/euler-works>



Part of the [Mathematics Commons](#)

Record Created:

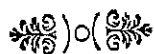
2018-09-25

Recommended Citation

Euler, Leonhard, "Methodus facilis computandi angulorum sinus ac tangentes tam naturales quam artificiales" (1750). *Euler Archive - All Works*. 128.

<https://scholarlycommons.pacific.edu/euler-works/128>

This Article is brought to you for free and open access by the Euler Archive at Scholarly Commons. It has been accepted for inclusion in Euler Archive - All Works by an authorized administrator of Scholarly Commons. For more information, please contact mgibney@pacific.edu.



METHODVS FACILIS COMPUTANDI ANGVLORVM SINVS AC TANGENTES TAM NATVRALES QVAM ARTIFICIALES

AVCTORE
Leonbardo Eulero.

§. 1.

Exposui anno praeterito methodum inueniendi valores eiusmodi expressionum, quae sint producta ex infinitis factoribus certa quadam lege progredientibus, eaque methodus deducta erat ex formulis integralibus, quarum integratio a se inuicem pendet. Nunc autem, cum nuper exposuisssem modum summamdi huiusmodi series

$$\frac{x}{1 \pm p} + \frac{x}{4 \pm p} + \frac{x}{9 \pm p} + \frac{x}{16 \pm p} + \frac{x}{25 \pm p} + \text{etc.}$$

ex eo nactus sum commodam atque aptam methodum quam plurimorum productorum, ex infinitis factoribus constantium, valores determinandi, eiusque beneficio mihi licuit innumerabiles istiusmodi expressiones definire, quae per alteram methodum vel omnino tractari non poterant, vel saltem tam expedite et concinne non absoluuntur. Quod negotium, quo clarius ob oculos ponatur, in sequentibus problematis sum complexurus.

Problema. 1.

§. 2. Inuenire valorem huius expressionis per continuos factores in infinitum progredientis.

+ +

$$\frac{1+p}{2}, \frac{1+p}{4}, \frac{1+p}{9}, \frac{1+p}{16}, \frac{1+p}{25}, \frac{1+p}{36}, \text{ etc.}$$

Solutio.

Ponatur huius expressionis propositae valor quaesitus $= s$,
et sumtis logarithmis, erit $ls = l(1+p) +$

$$l(1+\frac{p}{4}) + l(1+\frac{p}{9}) + l(1+\frac{p}{16}) + l(1+\frac{p}{25}) + l(1+\frac{p}{36}) + \text{etc.}$$

His igitur logarithmis per series notas expressis habebitur

$$ls = +\frac{p}{2} - \frac{p^2}{2} + \frac{p^3}{3} - \frac{p^4}{4} + \frac{p^5}{5} - \frac{p^6}{6} + \text{etc.}$$

$$+ \frac{p}{4} - \frac{p^2}{2 \cdot 4^2} + \frac{p^3}{3 \cdot 4^3} - \frac{p^4}{4 \cdot 4^4} + \frac{p^5}{5 \cdot 4^5} - \frac{p^6}{6 \cdot 4^6} + \text{etc.}$$

$$+ \frac{p}{9} - \frac{p^2}{2 \cdot 9^2} + \frac{p^3}{3 \cdot 9^3} - \frac{p^4}{4 \cdot 9^4} + \frac{p^5}{5 \cdot 9^5} - \frac{p^6}{6 \cdot 9^6} + \text{etc.}$$

$$+ \frac{p}{16} - \frac{p^2}{2 \cdot 16^2} + \frac{p^3}{3 \cdot 16^3} - \frac{p^4}{4 \cdot 16^4} + \frac{p^5}{5 \cdot 16^5} - \frac{p^6}{6 \cdot 16^6} + \text{etc.}$$

etc.

Sumantur differentia; eritque

$$\frac{ds}{dp} = 1 - p + p^2 - p^3 + p^4 - p^5 + \text{etc.}$$

$$+ \frac{1}{4} - \frac{p}{4^2} + \frac{p^2}{4^3} - \frac{p^3}{4^4} + \frac{p^4}{4^5} - \frac{p^5}{4^6} + \text{etc.}$$

$$+ \frac{1}{9} - \frac{p}{9^2} + \frac{p^2}{9^3} - \frac{p^3}{9^4} + \frac{p^4}{9^5} - \frac{p^5}{9^6} + \text{etc.}$$

$$+ \frac{1}{16} - \frac{p}{16^2} + \frac{p^2}{16^3} - \frac{p^3}{16^4} + \frac{p^4}{16^5} - \frac{p^5}{16^6} + \text{etc.}$$

etc.

Cum nunc hae series omnes sint geometricae, summari poterunt, hocque facto prodibit

$$\frac{ds}{dp} = \frac{1}{1+p} + \frac{1}{4+p} + \frac{1}{9+p} + \frac{1}{16+p} + \frac{1}{25+p} + \text{etc.}$$

Huius autem seriei summam nuper exhibui; unde si circuli cuius diameter $= 1$, peripheria ponatur $= \pi$ erit

$$\frac{ds}{dp} = \frac{\pi \sqrt{p-1}}{2p} + \frac{\pi \sqrt{p}}{p(e^{2\pi\sqrt{p-1}} - 1)} \quad \text{Ponatur facilitatis gratia } p =$$

qq , erit $dp = 2q dq$, atque aequatio inuenta abibit in hanc

B b 2

$$\frac{ds}{p} =$$

$$\frac{ds}{s} = \pi dq - \frac{dq}{q} + \frac{2\pi dq}{e^{2\pi q} - 1} = -\pi dq - \frac{dq}{q} + \frac{2e^{2\pi q} \pi dq}{e^{2\pi q} - 1}$$

Cuius integrale est $ls = lC - \pi q - lq + l(e^{2\pi q} - 1)$ seu

$$s = \frac{C(e^{2\pi q} - 1)}{e^{\pi l q}} = \frac{C(e^{2\pi \sqrt{p}} - 1)}{e^{\pi \sqrt{p} l \sqrt{p}}}, \text{ vbi constantem } C$$

ita determinari oportet, ut posito p vel $q = 0$ fiat $ls = 0$. At facto $q = 0$, fit $e^{2\pi q} - 1 = 2\pi q$, ideoque $ls = 0 = lC - \pi q - lq + l2\pi q = lC + l2\pi$, ergo $C = \frac{1}{2}\pi$. Consequenter expressionis propositae

$$\frac{1+p}{1} \cdot \frac{4+p}{4} \cdot \frac{9+p}{9} \cdot \frac{16+p}{16} \cdot \frac{25+p}{25} \cdot \frac{36+p}{36} \cdot \text{etc.}$$

$$\text{valor erit} = \frac{e^{2\pi \sqrt{p}} - 1}{2e^{\pi \sqrt{p}} \pi \sqrt{p}}. \quad \text{Q. E. I.}$$

Coroll. 1.

§. 3. Quodsi loco p ponatur $4p$, habebitur ista expressio:

$$\frac{1+4p}{1} \cdot \frac{4+4p}{4} \cdot \frac{9+4p}{9} \cdot \frac{16+4p}{16} \cdot \frac{25+4p}{25} \cdot \frac{36+4p}{36} \cdot \text{etc.}$$

$$\text{cuius igitur valor erit} \frac{e^{4\pi \sqrt{p}} - 1}{4e^{2\pi \sqrt{p}} \pi \sqrt{p}}.$$

Coroll. 2.

§. 4. Cum iam in hac expressione praecedens continueatur, diuidatur haec per illam, prodibitque

$$\frac{1+4p}{1} \cdot \frac{9+4p}{9} \cdot \frac{25+4p}{25} \cdot \frac{49+4p}{49} \cdot \text{etc.}$$

$$\text{cuius proinde valor est} \frac{e^{2\pi \sqrt{p}} + 1}{2e^{\pi \sqrt{p}}}.$$

Coroll.

Coroll. 3.

§. 5. Hinc igitur nanciscimur valorem huius expressionis propositae affinis :

$$\frac{1+p}{1} \cdot \frac{9+p}{9} \cdot \frac{25+p}{25} \cdot \frac{49+p}{49} \cdot \frac{81+p}{81} \cdot \text{etc.}$$

$$\text{quippe cuius valor erit} = \frac{e^{\pi\sqrt{p}} - 1}{2e^{\frac{1}{2}\pi\sqrt{p}}}.$$

Coroll. 4.

§. 6. Diuidatur per hanc ipsa expressio proposita, fiet

$$\frac{4+p}{4} \cdot \frac{16+p}{16} \cdot \frac{36+p}{36} \cdot \frac{64+p}{64} \cdot \frac{100+p}{100} \cdot \text{etc.}$$

$$\text{huius scilicet valor erit} = \frac{e^{\pi\sqrt{p}} - 1}{e^{\frac{1}{2}\pi\sqrt{p}} \pi\sqrt{p}}.$$

Coroll. 5.

§. 7. Si nunc expressio §. 5. per expressionem §. 6. diuidatur, prodibit haec forma

$$\frac{1+p}{3} \cdot \frac{4+p}{4+p} \cdot \frac{9+p}{9} \cdot \frac{16+p}{16+p} \cdot \frac{25+p}{25} \cdot \frac{36+p}{36+p} \cdot \text{etc.}$$

$$\text{cuius valor erit} = \frac{(e^{\pi\sqrt{p}} - 1)\pi\sqrt{p}}{2(e^{\pi\sqrt{p}} - 1)}.$$

Coroll. 6.

§. 8 Si sumantur binae huiusmodi series, atque altera per alteram diuidatur, obtinebuntur sequentes summationes.

$$\frac{1+p}{2+p} \cdot \frac{4+p}{4+p} \cdot \frac{9+p}{9+p} \cdot \frac{16+p}{16+p} \cdot \text{etc.} = \frac{e^{\pi\sqrt{q}}(e^{2\pi\sqrt{p}} - 1)\sqrt{q}}{e^{\pi\sqrt{p}}(e^{2\pi\sqrt{q}} - 1)\sqrt{p}}$$

B b 3

$$\frac{25+p}{25+p}$$

$$\frac{1+p}{1+q} \cdot \frac{1+q}{1+p} \cdot \frac{9+p}{9+q} \cdot \frac{16+q}{16+p} \cdot \text{etc.} = \frac{(e^{\pi\sqrt{p}}+1)(e^{\pi\sqrt{q}}-1)\sqrt{p}}{(e^{\pi\sqrt{p}}-1)(e^{\pi\sqrt{q}}+1)\sqrt{q}}$$

$$\frac{1+p}{1+q} \cdot \frac{9+p}{9+q} \cdot \frac{25+p}{25+q} \cdot \text{etc.} = \frac{e^{\frac{1}{2}\pi\sqrt{q}}(e^{\pi\sqrt{p}}+1)}{e^{\frac{1}{2}\pi\sqrt{p}}(e^{\pi\sqrt{q}}+1)}$$

$$\frac{1+p}{1+q} \cdot \frac{16+p}{16+q} \cdot \frac{36+p}{36+q} \cdot \text{etc.} = \frac{e^{\frac{1}{2}\pi\sqrt{q}}(e^{\pi\sqrt{p}}-1)\sqrt{q}}{e^{\frac{1}{2}\pi\sqrt{p}}(e^{\pi\sqrt{q}}-1)\sqrt{p}}.$$

§. 9. Ex solutione igitur huius primi problematis consequuti sumus valores eiusmodi productorum infinitis fractionibus contentorum, quarum tam numeratores quam denominatores sunt quadrata vel numerorum omnium in serie naturali prpgredientium, vel imparium tantum vel parium, eaque datis numeris aucta. Cum igitur istiusmodi factores in simplices reales, qui arithmetica teneant progressionem, resolui nequeant, istae summationes methedo iam ante exposita absolui non poterunt. At vicissim hinc non intelligitur, quinam prodituri sint valores, si vel p vel q negativae accipiantur ob exponentes $\pi\sqrt{p}$ et $\pi\sqrt{q}$, qui hoc casu fiunt imaginarii. Quamobrem hos casus in sequenti problemate euoluemus.

Problema 2.

§. 1. Inuenire valorem huius expressionis per continuos factores in infinitum progredientis

$$\frac{1-p}{1} \cdot \frac{4-p}{4} \cdot \frac{9-p}{9} \cdot \frac{16-p}{16} \cdot \frac{25-p}{25} \cdot \frac{36-p}{36} \cdot \text{etc.}$$

Solutio.

Ponatur valor quaesitus $= s$, eritque logarithmis sumendis,
 $\log s = \log(1-p) + \log(1-\frac{p}{4}) + \log(1-\frac{p}{9}) + \log(1-\frac{p}{16}) + \log(1-\frac{p}{25}) + \text{etc.}$
 His

His vero logarithmis in series conuerfis habebitur :

$$\begin{aligned} l s = & -\frac{p}{1} - \frac{p^2}{2} - \frac{p^3}{3} - \frac{p^4}{4} - \frac{p^5}{5} - \text{etc.} \\ & -\frac{p}{4} - \frac{p^2}{2 \cdot 4^2} - \frac{p^3}{3 \cdot 4^3} - \frac{p^4}{4 \cdot 4^4} - \frac{p^5}{5 \cdot 4^5} - \text{etc.} \\ & -\frac{p}{9} - \frac{p^2}{2 \cdot 9^2} - \frac{p^3}{3 \cdot 9^3} - \frac{p^4}{4 \cdot 9^4} - \frac{p^5}{5 \cdot 9^5} - \text{etc.} \\ & \text{etc.} \end{aligned}$$

Sumtisque differentialibus prodibit :

$$\begin{aligned} \frac{-ds}{sdp} = & +1 + p + p^2 + p^3 + p^4 + \text{etc.} \\ & +\frac{1}{4} + \frac{p}{4^2} + \frac{p^2}{4^3} + \frac{p^3}{4^4} + \frac{p^4}{4^5} + \text{etc.} \\ & +\frac{1}{9} + \frac{p}{9^2} + \frac{p^2}{9^3} + \frac{p^3}{9^4} + \frac{p^4}{9^5} + \text{etc.} \\ & \text{etc.} \end{aligned}$$

Quae series cum singulae sint geometricae, summae illarum loco substituantur, hincque erit

$$\frac{-ds}{sdp} = \frac{1}{1-p} + \frac{1}{4-p} + \frac{1}{9-p} + \frac{1}{16-p} + \frac{1}{25-p} + \text{etc.}$$

At istius seriei summam nuper elicui, quae, si substituitur, orietur

$$-\frac{ds}{sdp} = \frac{1}{2p} - \frac{\pi\sqrt{p}}{2p \tan A \pi\sqrt{p}}$$

Sit commodi ergo $p = qq$ eritque

$$-\frac{ds}{s} = \frac{dq}{q} - \frac{\pi dq}{\tan A \pi q} = \frac{dq}{q} - \frac{\pi dq \cos A \pi q}{\sin A \pi q}$$

Quoniam nunc est $d \cdot \sin A \cdot \pi q = \pi dq \cos A \cdot \pi q$, erit integrale aequationis inuentae,

$$lC - ls = lq - l \sin A \cdot \pi q; \text{ constante autem } C \text{ ita definita vt facto } p \text{ vel } q = 0 \text{ euanescat } ls \text{ prodibit } lC = lq - l \pi q = -l\pi. \text{ Quocirca erit } \frac{1}{\pi s} = \frac{q}{\sin A \cdot \pi q} = \frac{\sqrt{p}}{\sin A \cdot \pi \sqrt{p}}$$

$$\text{hincque } s = \frac{\sin A \cdot \pi \sqrt{p}}{\pi \sqrt{p}} \text{ siue}$$

$$\frac{1-p}{2} \cdot \frac{1-p}{4} \cdot \frac{9-p}{9} \cdot \frac{16-p}{16} \cdot \frac{25-p}{25} \cdot \text{etc.} = \frac{\sin A \cdot \pi \sqrt{p}}{\pi \sqrt{p}}.$$

Q. E. I.

Coroll. 1.

Coroll. 1.

§. 11. Quodsi loco p ponatur $4p$, habebitur ista expressio :

$$\frac{1-p}{1} \cdot \frac{1-p}{1} \cdot \frac{9-p}{9} \cdot \frac{4-p}{4} \cdot \frac{25-p}{25} \cdot \text{etc.}$$

$$\text{cuius valor erit} = \frac{\sin A \cdot 2\pi\sqrt{p}}{2\pi\sqrt{p}} = \frac{\sin A \cdot \pi\sqrt{p} \cdot \cos A \cdot \pi\sqrt{p}}{\pi\sqrt{p}}$$

Coroll. 2.

§. 12. Diuidatur haec series per illam, prodibitque

$$\frac{1-p}{1} \cdot \frac{9-p}{9} \cdot \frac{25-p}{25} \cdot \text{etc.} = \cos A \cdot \pi\sqrt{p} \text{ siue}$$

$$\frac{1-p}{1} \cdot \frac{9-p}{9} \cdot \frac{25-p}{25} \cdot \text{etc.} = \cos A \cdot \frac{\pi\sqrt{p}}{2}$$

Coroll. 3.

$$\text{§. 13. Cum iam sit } \frac{1-p}{1} \cdot \frac{4-p}{4} \cdot \frac{9-p}{9} \cdot \frac{16-p}{16} \cdot \text{etc.} =$$

$$\frac{\sin A \cdot \pi\sqrt{p}}{\pi\sqrt{p}} = \frac{2 \sin A \cdot \frac{1}{2} \pi\sqrt{p} \cdot \cos A \cdot \frac{1}{2} \pi\sqrt{p}}{\pi\sqrt{p}} \text{ erit}$$

$$\frac{4-p}{4} \cdot \frac{16-p}{16} \cdot \frac{36-p}{36} \cdot \frac{64-p}{64} = \frac{2 \sin A \cdot \frac{1}{2} \pi\sqrt{p}}{\pi\sqrt{p}}$$

Coroll. 4.

§. 14. Diuidatur per hanc expressionum praecedens oriatur.

$$\frac{1-p}{1} \cdot \frac{4-p}{4} \cdot \frac{9-p}{9} \cdot \frac{16-p}{16} \cdot \frac{25-p}{25} \cdot \frac{36-p}{36} \cdot \text{etc.}$$

$$\text{cuius valor erit} = \frac{\pi\sqrt{p}}{2 \tan A \cdot \frac{1}{2} \pi\sqrt{p}}$$

Coroll.

Coroll. 5.

§. 15. Si sumantur binæ huiusmodi sèries, earumque altera per alteram diuidatur, obtinebuntur sequentes summationes.

$$\frac{1-p}{1-q} \cdot \frac{4-p}{4-q} \cdot \frac{9-p}{9-q} \cdot \frac{16-p}{16-q} \text{ etc} = \frac{\sqrt{q} \sin. A. \pi \sqrt{p}}{\sqrt{p} \sin. A. \pi \sqrt{q}}$$

$$\frac{1-p}{1-q} \cdot \frac{4-q}{4-p} \cdot \frac{9-p}{9-q} \cdot \frac{16-q}{16-p} \text{ etc.} = \frac{\sqrt{p} \tan. A. \frac{1}{2} \pi \sqrt{q}}{\sqrt{q} \tan. A. \frac{1}{2} \pi \sqrt{p}}$$

$$\frac{1-p}{1-q} \cdot \frac{9-p}{9-q} \cdot \frac{25-p}{25-q} \cdot \frac{49-p}{49-q} \cdot \text{etc.} = \frac{\cos. A. \frac{1}{2} \pi \sqrt{p}}{\cos. A. \frac{1}{2} \pi \sqrt{q}}$$

$$\frac{4-p}{4-q} \cdot \frac{16-p}{16-q} \cdot \frac{36-p}{36-q} \cdot \frac{64-p}{64-q} \cdot \text{etc.} = \frac{\sqrt{q} \sin. A. \frac{1}{2} \pi \sqrt{p}}{\sqrt{p} \sin. A. \frac{1}{2} \pi \sqrt{q}}$$

§. 16. In his expressionibus sinus, cosinus et tangentes referuntur ad sinum totum = 1, seu arcus circulares in circulo sunt capiendi, cuius semidiameter est = 1. In tali igitur circulo exprimet π semissim peripheriae seu arcum 180. graduum. In numeris autem proximis erit, vt constat,

$$\pi = 3, 14159265357989$$

Quodsi vero \sqrt{p} et \sqrt{q} fuerint numeri rationales, tum sinus et tangentes, geometricè poterunt exhiberi, erit scilicet

$$\begin{array}{l} \sin. A. \pi = 0 \quad \left| \sin. A. \frac{1}{2} \pi = 1 \quad \left| \sin. A. \frac{1}{3} \pi = \frac{\sqrt{3}}{2} \quad \left| \sin. A. \frac{1}{4} \pi = \frac{1}{\sqrt{2}} \right. \right. \\ \cos. A. \pi = -1 \quad \left| \cos. A. \frac{1}{2} \pi = 0 \quad \left| \cos. A. \frac{1}{3} \pi = \frac{1}{2} \quad \left| \cos. A. \frac{1}{4} \pi = \frac{1}{\sqrt{2}} \right. \right. \\ \tan. A. \pi = 0 \quad \left| \tan. A. \frac{1}{2} \pi = \infty \quad \left| \tan. A. \frac{1}{3} \pi = \sqrt{3} \quad \left| \tan. A. \frac{1}{4} \pi = 1. \right. \right. \end{array}$$

§. 17. Expressionum harum vsus primum in hoc consistit, vt earum ope peripheria circuli multifariam

Tom. XI.

C c

per

per istiusmodi producta continua concinne possit exhiberi.
Quod vt appareat ponamus $p = \frac{m^2}{n^2}$ et cum π sit arcus
180. graduum erit per §. 10.

$$\frac{n^2-m^2}{n^2} \cdot \frac{4n^2-m^2}{4n^2} \cdot \frac{9n^2-m^2}{9n^2} \cdot \frac{16n^2-m^2}{16n^2} \text{ etc.} = \frac{n \sin. A. \frac{m}{n} 180^\circ}{m \pi}$$

$$\pi = \frac{n}{m} \sin. A. \frac{m}{n} 180^\circ \cdot \frac{n^2}{n^2-m^2} \cdot \frac{4n^2}{4n^2-m^2} \cdot \frac{9n^2}{9n^2-m^2} \cdot \frac{16n^2}{16n^2-m^2} \cdot \text{etc.}$$

unde emergunt sequentes pro valore ipsius π expressiones.

$$\text{Si } m = 1, n = 2$$

$$\pi = 2 \cdot \frac{4}{3} \cdot \frac{16}{15} \cdot \frac{36}{35} \cdot \frac{64}{63} \cdot \frac{100}{99} \cdot \frac{144}{143} \cdot \text{etc.}$$

$$\text{feu } \pi = 2 \cdot \frac{2 \cdot 2 \cdot 4 \cdot 4 \cdot 6 \cdot 6 \cdot 8 \cdot 8 \cdot 10 \cdot 10 \cdot 12 \cdot 12}{1 \cdot 3 \cdot 3 \cdot 5 \cdot 5 \cdot 7 \cdot 7 \cdot 9 \cdot 9 \cdot 11 \cdot 11 \cdot 13} \text{ etc.}$$

quae est ipsa expressio Wallisii alibi a me demonstrata.

$$\text{Si } m = 1 \text{ et } n = 3$$

$$\pi = \frac{3\sqrt{3}}{2} \cdot \frac{9}{8} \cdot \frac{36}{35} \cdot \frac{81}{80} \cdot \frac{144}{143} \cdot \frac{225}{224} \text{ etc. feu}$$

$$\pi = \frac{3\sqrt{3}}{2} \cdot \frac{3 \cdot 3 \cdot 6 \cdot 6 \cdot 9 \cdot 9 \cdot 12 \cdot 12 \cdot 15 \cdot 15}{2 \cdot 4 \cdot 5 \cdot 7 \cdot 8 \cdot 10 \cdot 11 \cdot 13 \cdot 14 \cdot 16} \text{ etc.}$$

$$\text{Si } m = 1 \text{ etc. } n = 4$$

$$\pi = 2\sqrt{2} \cdot \frac{16}{15} \cdot \frac{64}{63} \cdot \frac{144}{143} \cdot \frac{256}{255} \cdot \text{etc. feu}$$

$$\pi = 2\sqrt{2} \cdot \frac{4 \cdot 4 \cdot 8 \cdot 8 \cdot 12 \cdot 12 \cdot 16 \cdot 16 \cdot 20 \cdot 20}{3 \cdot 5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 15 \cdot 17 \cdot 19 \cdot 21} \text{ etc.}$$

$$\text{Si } m = 1 \text{ et } n = 6$$

$$\pi = 3 \cdot \frac{36}{35} \cdot \frac{144}{143} \cdot \frac{524}{523} \cdot \frac{576}{575} \text{ etc. feu}$$

$$\pi = 3 \cdot \frac{6 \cdot 6 \cdot 12 \cdot 12 \cdot 18 \cdot 18 \cdot 24 \cdot 24}{5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 25} \text{ etc.}$$

§. 18. Expressiones hae, quanquam satis cito convergunt, tamen sunt aptiores ad logarithmum ipsius π inueniendum, quam ad ipsum valorem π . Ita erit ex vltima expressione

$$l \pi = l 3 + l \frac{36}{35} + l \frac{144}{143} + l \frac{524}{523} + \text{etc. feu}$$

$$l \pi = l 3 + \frac{1}{62} \left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \text{etc.} \right)$$

$$+ \frac{1}{2 \cdot 64} \left(1 + \frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \frac{1}{25^2} + \text{etc.} \right)$$

+

$$+ \frac{1}{3 \cdot 60} \left(1 + \frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \frac{1}{25^3} + \text{etc.} \right)$$

$$+ \frac{1}{4 \cdot 6^3} \left(1 + \frac{1}{4^4} + \frac{1}{9^4} + \frac{1}{16^4} + \frac{1}{25^4} + \text{etc.} \right)$$

etc.

vnde calculus sequenti modo instituetur ad logarithmum hyperbolicum ipsius π inueniendum

$$l 3 = 1,098612288668$$

$$l \frac{36}{35} = 0,028170876966$$

$$0,017914835217 = \frac{1}{8^2} \left(\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \text{etc.} \right)$$

$$0,000031760507 = \frac{1}{2 \cdot 6^2} \left(\frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \text{etc.} \right)$$

$$0,000000123907 = \frac{1}{3 \cdot 6^3} \left(\frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \text{etc.} \right)$$

$$0,000000000607 = \frac{1}{4 \cdot 6^4} \left(\frac{1}{4^4} + \frac{1}{9^4} + \frac{1}{16^4} + \text{etc.} \right)$$

4

$$l \pi = 1,144729885879$$

Logarithmus hic hyperbolicus si multiplicetur. per

$$0,434294481903251$$

prodibit logarithmus communis valoris π seu numeri

$$3,14159265357989 \text{ etc.}$$

qui logarithmus a Cl. Sharpio in Tabulis mathematicis computatus est

$$0,49714,98726,94133,85435,12682,88290. \text{ etc.}$$

§. 19. Cum autem peripheria circuli per se satis sit cognita ex approximationibus iam diligentissime peractis, vsui harum expressionum in hoc negotio supersedebimus. Alter autem vsus, qui ex his expressionibus duci potest, consistit in inueniendis sinibus et tangentibus et secantibus quorumcunque angulorum, qua quidem in re opus est

nosse valorem ipsius π . Ita si ponamus $\pi = 2q$ ita ut sit q arcus 90 graduum erit

$$\sin. A. \frac{m}{n} q = \frac{m}{n} q \cdot \frac{4n^2 - m^2}{4n^2} \cdot \frac{16n^2 - m^2}{16n^2} \cdot \frac{36n^2 - m^2}{36n^2} \cdot \frac{64n^2 - m^2}{64n^2} \text{ etc.}$$

hincque

$$\cos. A. \frac{m}{n} q = \frac{n}{m} q \cdot \frac{4n^2}{4n^2 - m^2} \cdot \frac{16n^2}{16n^2 - m^2} \cdot \frac{36n^2}{36n^2 - m^2} \cdot \frac{64n^2}{64n^2 - m^2} \text{ etc.}$$

Porro ex §. 12. posito $\sqrt{p} = \frac{m}{n}$ habebitur

$$\cos. A. \frac{m}{n} q = \frac{n^2 - m^2}{n^2} \cdot \frac{5n^2 - m^2}{5n^2} \cdot \frac{25n^2 - m^2}{25n^2} \cdot \frac{49n^2 - m^2}{49n^2} \text{ etc.}$$

hincque

$$\sec. A. \frac{m}{n} q = \frac{n^2}{n^2 - m^2} \cdot \frac{5n^2}{5n^2 - m^2} \cdot \frac{25n^2}{25n^2 - m^2} \cdot \frac{49n^2}{49n^2 - m^2} \text{ etc.}$$

Denique ex §. 14. deducitur pari modo

$$\tan. A. \frac{m}{n} q = \frac{m}{n} q \cdot \frac{n^2}{n^2 - m^2} \cdot \frac{4n^2}{4n^2 - m^2} \cdot \frac{9n^2}{9n^2 - m^2} \cdot \frac{16n^2}{16n^2 - m^2} \text{ etc.}$$

hincque

$$\cot. A. \frac{m}{n} q = \frac{n}{mq} \cdot \frac{n^2 - m^2}{n^2} \cdot \frac{4n^2}{4n^2 - m^2} \cdot \frac{9n^2 - m^2}{9n^2} \cdot \frac{16n^2}{16n^2 - m^2} \text{ etc.}$$

Hae vero formulae, etsi vehementer conuergunt, tamen multo sunt aptiores ad logarithmos finuum, tangentium et secantium inueniendos; quem usum singularem antiquam exponamus, methodum facilem aperiemus, ipsos sinus et tangentes expedite computandi: idque sine coniectis subsidiis ex multiplicatione arcuum, aliisque huc pertinentibus theorematibus.

Problema 3.

§. 20. Inuenire canonem generalem, ad sinus et cosinus angulorum quorumcunque inueniendos idoneum.

Solutio.

Formulae, quas hic pro sinibus et cosinibus exhibuimus, si euoluantur, recidunt ad formulas iam pridem notas; scilicet posito arcu circuli $= s$, fit

$\sin. A$

$$\sin. A. s = s - \frac{s^3}{1 \cdot 2 \cdot 3} + \frac{s^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \frac{s^7}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdot 7} + \text{etc.}$$

$$\cos. A. s = 1 - \frac{s^2}{1 \cdot 2} + \frac{s^4}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{s^6}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \text{etc.}$$

posito sinu toto = 1. Quodsi ergo ponatur q pro arcu
90. graduum, sumaturque arcus propositus $s = \frac{m}{n} q$, fiet

$$\sin. A. \frac{m}{n} q = \frac{m}{n} \cdot q - \frac{m^3}{n^3} \cdot \frac{q^3}{1 \cdot 2 \cdot 3} + \frac{m^5}{n^5} \cdot \frac{q^5}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5} - \text{etc.}$$

$$\cos. A. \frac{m}{n} q = 1 - \frac{m^2}{n^2} \cdot \frac{q^2}{1 \cdot 2} + \frac{m^4}{n^4} \cdot \frac{q^4}{1 \cdot 2 \cdot 3 \cdot 4} - \text{etc.}$$

Cum igitur sit $q = \frac{\pi}{2}$ erit

$$q = 1, 570796326794896619231313216916$$

Hoc vero valore loco potestatum ipsius q computato ac
substituto, obtinebuntur formulae numericae, quibus tam
sinus quam cosinus arcus $\frac{m}{n} q$ exprimentur. Quoniam vero
tantum pro arcubus 45° minoribus sinus et cosinus desi-
derantur erit $\frac{m}{n} < \frac{1}{2}$, et hanc ob rem series datae maxime
conuergent. Supputavi ego autem singulos h. rum serie-
rum terminos a solo q pendentes in fractionibus decimali-
bus ad 28. figuras, quas, vt. alios calculo tam taedioso
liberem, hic appono.

Erit igitur sinus arcus $\frac{m}{n} 90$ graduum =

$$\begin{aligned} & + \frac{m}{n} \cdot 1, 5707963267948966192313216916 \\ & - \frac{m^3}{n^3} \cdot 0, 6459640975062462536557565636 \\ & + \frac{m^5}{n^5} \cdot 0, 0796926262461670451205055487 \\ & - \frac{m^7}{n^7} \cdot 0, 0046817541353186881006854633 \\ & + \frac{m^9}{n^9} \cdot 0, 0001604411847873598218726605 \\ & - \frac{m^{11}}{n^{11}} \cdot 0, 0000035988432352120853404580 \\ & + \frac{m^{13}}{n^{13}} \cdot 0, 0000000569217292196792681170 \\ & - \frac{m^{15}}{n^{15}} \cdot 0, 0000000006688035109811467225 \end{aligned}$$

C c 3

+

+	$\frac{m^0}{n^0}$. I, 00000000000000000000000000000000
-	$\frac{m^2}{n^2}$. I, 2337005501361698273543113745
+	$\frac{m^4}{n^4}$. 0, 2536695079010480136365633659
-	$\frac{m^6}{n^6}$. 0, 0208634807633529608730516364
+	$\frac{m^8}{n^8}$. 0, 0009192602748394265802417158
-	$\frac{m^{10}}{n^{10}}$. 0, 0000252020423730606054810526
+	$\frac{m^{12}}{n^{12}}$. 0, 0000004710874778818171503665
-	$\frac{m^{14}}{n^{14}}$. 0, 0000000063866030837918522408
+	$\frac{m^{16}}{n^{16}}$. 0, 0000000000656596311497947230
-	$\frac{m^{18}}{n^{18}}$. 0, 00000000000005294400200734620
+	$\frac{m^{20}}{n^{20}}$. 0, 000000000000000034377391790981
-	$\frac{m^{22}}{n^{22}}$. 0, 0000000000000000000000183599165212
+	$\frac{m^{24}}{n^{24}}$. 0, 00000000000000000000000820675327
-	$\frac{m^{26}}{n^{26}}$. 0, 0000000000000000000000003115285
+	$\frac{m^{28}}{n^{28}}$. 0, 0000000000000000000000000010165
-	$\frac{m^{30}}{n^{30}}$. 0, 000000000000000000000000000026

Quo-

Quocunque igitur angulo proposito, eius ratio ad 90° est primum quaerenda, quae sit vt m ad n , qua inuenta, si in his formulis fiat substitutio debito modo, reperietur tam sinus quam cosinus anguli propositi.

Q. E. I.

§. 21. Quodsi igitur ponatur $\frac{m}{n} = 1$, prior formula dare debet sinum totum $= 1$, quod vt appareat calculum subiiciamus.

+	1, 5707963267948956192313216916
-	0, 6459640975062462536557565636
<hr/>	
+	0, 9248322292886503655755651280
+	0, 0796926262461670451205055487
<hr/>	
+	1, 0045248555348174106960706767
-	46817541353186881006854633
<hr/>	
+	0, 9998431013994987225953852134
+	1604411847873598218726605
<hr/>	
+	1, 0000035425842860824172578739
-	35988432352120853404580
<hr/>	
+	0, 9999999437410508703319174159
+	569217292196792681170
<hr/>	
+	1, 0000000006627800900111855329
-	6688035109811467225
<hr/>	
+	0, 9999999999939765790300388104
+	60669357311061950
<hr/>	
+	1, 0000000000000435147611450054
-	437706546731370
<hr/>	

+

[illegible][illegible]

§. 22. Exemplum hoc adieci, ut appareat in computo harum formularum errorem a me non esse commissum, easque ideo tuto adhiberi posse. Quod idem ut clarius perspiciatur, calculum etiam cosinus anguli 90° hic apponam, qui debet esse $= 0$.

+ I, oooooooooooooooooooooo
- I, 233700550I361698273543II3745

— 0, 2337C0550I361698273543II3745

+ 2536695079010480126365633659

+ 199689577048781862822519914
 - 208634807633529608730516364

-	8945229984747745907996450
+	9192602748394265802417158
<hr/>	
+	247372763646519894420708
-	252020423730606054810526
<hr/>	
-	4647660084086100389818
+	4710874778818171503665
<hr/>	
+	63214694732011113847
-	63866030837918522408
<hr/>	
-	651330105907408561
+	656596311497947230
<hr/>	
+	5260205590538669
-	5294400200734620
<hr/>	
-	34194010195951
+	34377391790981
<hr/>	
+	182781595030
-	183599165212
<hr/>	
-	817570182
+	820675327
<hr/>	
+	3105145
-	3115285
<hr/>	
-	10140
+	10165
<hr/>	
+	25
-	26
<hr/>	
-	1

210 METHOD. FACIL. COMPVT. ANGV. SINVS

Qui consensu cum veritate tantus est, vt de veritate datarum formularum dubitare amplius non liceat.

§. 23. Quaeramus speciminis loco finum et cosinum anguli g graduum, qui casus est facilis ob valorem $\frac{m}{n} = \frac{1}{12}$. Ac primo quidem pro finu erunt termini affirmatiui

0, 1570796326794896619231321691
7969262624616704502050
1604411847873598
56921729

+ 0, 1570804290059125647840629069

Termini vero negatiui sunt.

0, 0006459640975062462536557565
4681754135318688100
359884323521
6688

- 0, 0006459645656816957739575874

+ 0, 1570804296059125647840629069

0, 1564344650402308690101053195 = fin. 9°

Pro cosinu autem sunt termini affirmatiui

1, 000000000000000000000000000000
253669507901048013636563
91926027483942658
4710874778
65

+ 1, 0000253669599827080208454065

Termini vero negatiui

0,

$$\begin{array}{r}
0,0123370055013616982735431137 \\
208634807633529608730 \\
25202042373060 \\
638660
\end{array}$$

$$\begin{array}{r}
- 0,0123370263648449818308051587 \\
+ 1,0000253669599827080208454065
\end{array}$$

$0,9876383405951377261900402478 = \cos. 9^\circ$
 Hoc autem exemplum, etsi in suo genere est facillimum, ramen abunde declarat utilitatem formularum datarum, atque compendium, quod illae calculo alias operosissimo afferunt.

§. 24. Labor autem istius computi multo fiet minor, si finus et cosinus non ad tot figuras in fractionibus decimalibus desiderentur. Ponamus igitur sinum totum seu radium esse

10000000000

atque pro hoc radio erit

$$\begin{array}{rcl}
\sin. A. \frac{m}{n} 90^\circ & = & + \frac{m}{n} \cdot 15707963267,94 \\
& - & \frac{m^3}{n^3} \cdot 6459640975,06 \\
& + & \frac{m^5}{n^5} \cdot 796926162,46 \\
& - & \frac{m^7}{n^7} \cdot 46817541,35 \\
& + & \frac{m^9}{n^9} \cdot 1604411,84 \\
& - & \frac{m^{11}}{n^{11}} \cdot 35988,43 \\
& + & \frac{m^{13}}{n^{13}} \cdot 569,21 \\
& - & \frac{m^{15}}{n^{15}} \cdot 6,68 \\
& + & \frac{m^{17}}{n^{17}} \cdot 0,06
\end{array}$$

Atque pari modo erit

D d 2

cos.

$$\begin{aligned}
\cos. A. \frac{m}{n} 90^\circ &= + 10000000000, 00 \\
&- \frac{m^2}{n^2} 12337005501, 36 \\
&+ \frac{m^4}{n^4} .2536695079, 01 \\
&- \frac{m^6}{n^6} ..208634807, 63 \\
&+ \frac{m^8}{n^8} ... 9192602, 74 \\
&- \frac{m^{10}}{n^{10}} 252020, 42 \\
&+ \frac{m^{12}}{n^{12}} 4710, 87 \\
&- \frac{m^{14}}{n^{14}} 63, 86 \\
&+ \frac{m^{16}}{n^{16}} , 65
\end{aligned}$$

vbi partes centesimas adiecimus, vt de vltimis figuris p-
nitus certi esse queamus.

Problema 4.

§. 25. Inuenire canonem generalem pro inueniendis
tangentibus et cotangentibus omnium angulorum.

Solutio.

Quod primum ad tangentes attinet, ponatur angulus rec-
tus seu $90^\circ = q$, propositusque fit angulus $\frac{m}{n}q$ graduum
seu $\frac{m}{n} 90^\circ$, erit posito sinu toto $= 1$, tang. A. $\frac{m}{n} 90^\circ$
 $= \frac{2m}{nq} \left(\frac{n^2}{n^2-m^2} + \frac{n^2}{9n^2-m^2} + \frac{n^2}{25n^2-m^2} + \text{etc.} \right)$

$$\begin{aligned}
\text{tang. A. } \frac{m}{n} 90^\circ &= \frac{2m}{nq} \left(1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \text{etc.} \right) \\
&+ \frac{2m^3}{n^3q} \left(1 + \frac{1}{3^4} + \frac{1}{5^4} + \frac{1}{7^4} + \text{etc.} \right) \\
&+ \frac{2m^5}{n^5q} \left(1 + \frac{1}{3^6} + \frac{1}{5^6} + \frac{1}{7^6} + \text{etc.} \right) \\
&+ \frac{2m^7}{n^7q} \left(1 + \frac{1}{3^8} + \frac{1}{5^8} + \frac{1}{7^8} + \text{etc.} \right) \\
&\text{etc.}
\end{aligned}$$

Quodsi

Quodsi autem termini primi harum ferierum seorsim capiuntur, vt reliqui eo magis conuergant erit

$$\begin{aligned} \text{tang. A. } \frac{m}{n} 90^\circ = & + \frac{2mn}{(nn-mm)q} \\ & + \frac{2m}{nq} \left(\frac{1}{3} 2 + \frac{1}{5} 2 + \frac{1}{7} 2 + \text{etc.} \right. \\ & + \frac{2m^3}{3n^3} \left(\frac{1}{5} 4 + \frac{1}{7} 4 + \frac{1}{9} 4 + \text{etc.} \right. \\ & + \frac{2m^5}{15n^5} \left(\frac{1}{7} 6 + \frac{1}{9} 6 + \frac{1}{11} 6 + \text{etc.} \right. \\ & + \frac{2m^7}{n^7} \left(\frac{1}{9} 8 + \frac{1}{11} 8 + \frac{1}{13} 8 + \text{etc.} \right. \\ & \text{etc.} \end{aligned}$$

Est vero $\frac{1}{n} = \frac{1}{2} q = 0, 31830988618379$

hincque $\frac{2}{q} = 1, 27323954473516$. Quare si summae istarum ferierum quae proxime habentur, per hunc valorem multiplicentur prodibit

$$\begin{aligned} \text{tang. A. } \frac{m}{n} 90^\circ = & + \frac{m}{n-m} . 0, 6366197723675 \\ & + \frac{m}{n+m} . 0, 6366197723675 \\ & + \frac{m}{n} . 0, 2975567820597 \\ & + \frac{m^3}{n^3} . 0, 0186886502773 \\ & + \frac{m^5}{n^5} . 0, 0018424752034 \\ & + \frac{m^7}{n^7} . 0, 0001975800714 \\ & + \frac{m^9}{n^9} . 0, 0000216977245 \\ & + \frac{m^{11}}{n^{11}} . 0, 0000024011370 \\ & + \frac{m^{13}}{n^{13}} . 0, 0000002664132 \\ & + \frac{m^{15}}{n^{15}} . 0, 0000000295864 \\ & + \frac{m^{17}}{n^{17}} . 0, 0000000032867 \\ & + \frac{m^{19}}{n^{19}} . 0, 0000000003651 \\ & + \frac{m^{21}}{n^{21}} . 0, 0000000000405 \\ & + \frac{m^{23}}{n^{23}} . 0, 0000000000045 \\ & + \frac{m^{25}}{n^{25}} . 0, 0000000000000 \end{aligned}$$

D d 3

cuius

cuius formulae ope tangentes in fractionibus decimalibus ad 12. figuras facile computari poterunt posito sinu toto = 1.

Quod secundo ad cotangentes attinet, erit iisdem positis

$$\cotang. A. \cdot \frac{m}{n} 90^\circ = \frac{n}{mq} - \frac{m}{2nq} \left(\frac{4n^2}{4n^2-m^2} + \frac{4n^2}{16n^2-m^2} + \frac{4n^2}{36n^2-m^2} + \frac{4n^2}{64n^2-m^2} + \text{etc.} \right)$$
 seu

$$\cot. A. \cdot \frac{m}{n} 90^\circ = \frac{n}{mq} - \frac{m}{2nq} \left(1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \text{etc.} \right)$$

$$- \frac{m^3}{8n^3q} \left(1 + \frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \text{etc.} \right)$$

$$- \frac{m^5}{32n^5q} \left(1 + \frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \text{etc.} \right)$$

$$- \frac{m^7}{128n^7q} \left(1 + \frac{1}{4^4} + \frac{1}{9^4} + \frac{1}{16^4} + \text{etc.} \right)$$
 etc.

Additis autem terminis primis erit

$$\cot. A. \cdot \frac{m}{n} 90^\circ = \frac{n}{mq} - \frac{m}{2n-m} \cdot \frac{1}{2q} - \frac{m}{2n+m} \cdot \frac{1}{2q}$$

$$- \frac{2m}{nq} \cdot \frac{1}{4} \left(\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \text{etc.} \right)$$

$$- \frac{2m^3}{n^3q} \cdot \frac{1}{4^2} \left(\frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \frac{1}{25^2} + \text{etc.} \right)$$

$$- \frac{2m^5}{n^5q} \cdot \frac{1}{4^3} \left(\frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \frac{1}{25^3} + \text{etc.} \right)$$
 etc.

At tam serierum loco summis substituendis, quam loco q valore debito, obtinebitur

$$\cot. A. \cdot \frac{m}{n} q = + \frac{n}{m} \cdot 0, 6366197723675$$

$$- \frac{m}{2n-m} \cdot 0, 3183098861837$$

$$- \frac{m}{2n+m} \cdot 0, 3183098861837$$

$$- \frac{m}{n} \cdot 0, 2052888894145$$

$$- \frac{m^3}{n^3} \cdot 0, 0065510747882$$

$$- \frac{m^5}{n^5} \cdot 0, 0003450292554$$

$$- \frac{m^7}{n^7} \cdot 0, 0000202791060$$

$$- \frac{m^9}{n^9} \cdot 0, 0000012366527$$

$$\begin{aligned}
 & - \frac{m^{11}}{n^{11}} . 0, 00000000764959 \\
 & - \frac{m^{13}}{n^{13}} . 0, 0000000047597 \\
 & - \frac{m^{15}}{n^{15}} . 0, 0000000002969 \\
 & - \frac{m^{17}}{n^{17}} . 0, 0000000000186 \\
 & - \frac{m^{19}}{n^{19}} . 0, 00000000000011
 \end{aligned}$$

Huiusque formulae ope cotangentes angulorum omnium 90. gradibus minorum expedite reperiri poterunt.

Q. E. I.

§. 26. Quanquam ex datis anguli finu et cofinu eiusdem tangens et cotangens inueniri possunt, tamen diuisio, quae adhiberi debet, plerumque nimis molesta esse solet. Quamobrem formulas hic datas multo aptiores esse merito arbitramur ad tangentes et cotangentes quorumuis angulorum inueniendas. Vt autem veritas harum regularum perspiciatur, eiusmodi exempla tangentium et cotangentium euoluamus, quae per se sint cognita. Quaeratur itaque tangens anguli semirecti seu 45° , quam constat esse aequalem finui toti seu 1. Erit igitur $m=1$ et $n=2$: vnde termini prodibunt sequentes addendi:

$$\begin{array}{r}
 0, 6366197723675 \\
 0, 2122065907891 \\
 1487783910298 \\
 23360812847 \\
 575773501 \\
 15435943 \\
 423784 \\
 11724 \\
 325 \\
 0 \\
 \hline
 1, 0000000000000
 \end{array}$$

vti

vbi in additione vltimae columnae tres vnitates sunt adiectae, quippe quae proditurae fuisse censendae sunt ex sequentibus columnis, si affuissent. Ceterum ex formula manifestum est tangentem anguli recti fore infinitam ob $n-m=0$. Pro cotangente sumamus exemplum anguli recti, cuius cotangens est $=0$. Cum igitur expressio nostra cotangentis omnes terminos praeter primum habeat negativos addamus terminos negativos seorsim, qui ob $m=n=1$ ita se habebunt.

0, 3183098861837

0, 1061032953946

0, 2052888894145

65510747882

3450292554

202791060

12366527

764959

47597

2969

186

11

0, 6366197723675

Terminus autem affirmatiuus, a quo haec summa auferri debet est

0, 6366197723675

ita vt cotangens anguli recti actu reperiatur $=0$.

§. 27. Etsi autem haec, quae de inuentione finium et tangentium attulimus ex seriebus meis nuper expositis consequuntur, tamen eadem hae formulae ex aliis iam dudum cognitis seriebus deduci potuissent. His igitur relictis

liſtis progredior ad ea, quae huic methodo ſummandi ſeries ſunt propria, atque modum docebo facilem inueniendi logarithmos ſinuum, et tangentium quorumcunque angulorum; qui eo magis eſt notatu dignus, quod logarithmos ſive ſinuum ſive tangentium praebeat, ſine praeterea ipſorum ſinuum ac tangentium cognitione. Cum autem logarithmi ſint duplices, vel naturales ſeu hyperbolici, vel decadici, in quibus logarithmus 10 ponitur = 1, vtriusque generis logarithmos hic inuenire docebo.

Problema. 5.

§ 28. Definire logarithmum tam naturalem quam conſuetum ſive ſinus ſive coſinus anguli cuiuscunque poſiti.

Solutio.

Ex paragr. 19. capiatur pro logarithmo ſinus inueniendo expreſſio haec

$$\sin. A. \frac{m}{n} q = \frac{m}{n} q \cdot \frac{n^2 - m^2}{4n^2} \cdot \frac{16n^2 - m^2}{16n^2} \cdot \frac{36n^2 - m^2}{36n^2} \cdot \text{etc.}$$

quae in logarithmos conuerſa ſtatim dat

$$l \sin. A. \frac{m}{n} q = l \frac{m}{n} + (1 - \frac{m^2}{4n^2}) + l(1 - \frac{m^2}{16n^2}) + \text{etc.}$$

Quaeratur primo logarithmus naturalis ſinus anguli $\frac{m}{n} q$ ſeu $\frac{m}{n} 90^\circ$, eritque logarithmis per ſeries expreſſis

$$\begin{aligned} l \sin. A. \frac{m}{n} q &= l q + l \frac{m}{n} \\ &\quad - \frac{m^2}{4n^2} (1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \text{etc.}) \\ &\quad - \frac{m^4}{2 \cdot 16n^4} (1 + \frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \text{etc.}) \\ &\quad - \frac{m^6}{8 \cdot 16n^6} (1 + \frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \text{etc.}) \\ &\quad - \frac{m^8}{4 \cdot 16n^8} (1 + \frac{1}{4^4} + \frac{1}{9^4} + \frac{1}{16^4} + \text{etc.}) \\ &\quad \text{etc.} \end{aligned}$$

sive $l \sin. A \frac{m}{n} q = l q - l \frac{n}{m} - l \frac{4n^2}{4n^2 - m^2}$
 $- \frac{m^2}{1.3n^2} \left(\frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \frac{1}{25} + \text{etc.} \right)$
 $- \frac{m^4}{2.4.2n^4} \left(\frac{1}{4^2} + \frac{1}{9^2} + \frac{1}{16^2} + \frac{1}{25^2} + \text{etc.} \right)$
 $- \frac{m^6}{3.1.3.1n^6} \left(\frac{1}{4^3} + \frac{1}{9^3} + \frac{1}{16^3} + \frac{1}{25^3} + \text{etc.} \right)$
 $- \frac{m^8}{4.4.4n^8} \left(\frac{1}{4^4} + \frac{1}{9^4} + \frac{1}{16^4} + \frac{1}{25^4} + \text{etc.} \right)$
 etc.

Quodsi nunc loco harum serierum summae proximae substituantur, eaeque per coefficientes numericas multiplicentur prodibit $l \sin. A \frac{m}{n} 90^\circ =$

$$l q - 2 l 2 - 3 l n + l m + l(2n - m) + l(2n + m)$$

$$- \frac{m^2}{n^2} . 0, 16123351671205660911810379$$

$$- \frac{m^4}{n^4} . 0, 00257260105347306848487511$$

$$- \frac{m^6}{n^6} . 0, 00009032844783567260267978$$

$$- \frac{m^8}{n^8} . 0, 00000398179316205501892449$$

$$- \frac{m^{10}}{n^{10}} . 0, 00000019425295465196979631$$

$$- \frac{m^{12}}{n^{12}} . 0, 00000001001328748812045486$$

$$- \frac{m^{14}}{n^{14}} . 0, 00000000053404135618987888$$

$$- \frac{m^{16}}{n^{16}} . 0, 00000000002914859658937808$$

$$- \frac{m^{18}}{n^{18}} . 0, 00000000000161797979778706$$

$$- \frac{m^{20}}{n^{20}} . 0, 00000000000009097690905311$$

$$- \frac{m^{22}}{n^{22}} . 0, 00000000000000051682754587$$

$$- \frac{m^{24}}{n^{24}} . 0, 00000000000000002960770778$$

$$- \frac{m^{26}}{n^{26}} . 0, 00000000000000000170813$$

$$- \frac{m^{28}}{n^{28}} . 0, 00000000000000000009913$$

$$- \frac{m^{30}}{n^{30}} . 0, 00000000000000000000578$$

$$- \frac{m^{32}}{n^{32}} . 0, 00000000000000000000034$$

qui

qui termini postemi etfi non eousque sint continuati ac priores, tamen potestate vltius porriguntur nisi sit $\frac{m}{n} = 1$. Dat autem haec forma logarithmum hyperbolicum sinus anguli $\frac{m}{n} 90^\circ$, posito sinu toto $= 1$ eiusque logarithmo $= 0$. Debent autem pro hoc negotio etiam numerorum $2, n, m, 2n - m$ et $2n + m$ logarithmi hyperbolici accipi, itemque ipsius q , quem supra indicauimus. Hac vero ipsa methodo poterit lq accuratius exhiberi. Quodsi enim ponatur $m = 1$ et $n = 2$ erit $l \sin. A. 45' = l \frac{1}{\sqrt{2}} = -\frac{1}{2} l 2$: ferierum vero summae additae conficient vt sequitur.

0, 04030837917801415227952595
 16078756584206678030469
 141138199743238441687
 1555387953927741767
 18970015102731425
 244465026565441
 3259529761901
 44477228683
 617210311
 8676234
 123221
 1765
 25

0, 04047059387191103834465527

qui valor ponatur tantisper $= a$ eritque $lq = a + 5 l 2 - \frac{1}{2} l 2 - l 3 - l 5$ est vero

$$\frac{2}{3}l2 = 3, 11916231251975389237754454$$

$$l3 = 1, 09861228866810969139524526$$

$$l5 = 1, 60943791243410037460075935$$

$$\frac{2}{3}l2 - l15 = 0, 41111211141754382638153993$$

$$\alpha = 0, 04047059387191103834465527$$

$$lq = 0, 45158270528945486472619520$$

$$l2 = 0, 69314718055994530941723212$$

$$l2q = 1, 14472988584940017414342732$$

qui est valor pro logarithmo hyperbolico ipsius π , quem supra minus accurate §. 18. definiuimus. Quare si iste valor loco lq substituatur, facili negotio logarithmi hyperbolici finium quorumvis angulorum reperiri poterunt, vbi hoc tantum est monendum, numerorum $2, m, n, 2n - m$ et $2n + m$ logarithmos quoque hyperbolicos sumi debere; qui vel facile computantur vel passim computati reperiuntur. Ex logarithmis autem hyperbolicis inueniuntur logarithmi communes, si illi multiplicentur per

$$0, 4342944819325182$$

Fiat igitur haec multiplicatio, et tum addatur 10, eo quod in tabulis ordinariis logarithmus sinus totius poni solet = 10, quo facto erit

$$\log. \sin. A. \frac{m}{n} 90^\circ = l(2n + m) + l(1n - m) - lm - 3ln$$

$$+ 9, 59405988570218017$$

$$- \frac{m^2}{n^2} . 0, 07002282660590191$$

$$- \frac{m^4}{n^4} . 0, 00111726644166184$$

$$- \frac{m^6}{n^6} . 0, 00003922914645391$$

$$- \frac{m^8}{n^8} . 0, 00000172927079836$$

$$- \frac{m^{10}}{n^{10}} . 0, 00000008436298629$$

$$- \frac{m^{12}}{n^{12}} . 0, 00000000434871550$$

$$\begin{aligned}
-\frac{m^{14}}{n^{14}} &. 0, 00000000023193121 \\
-\frac{m^{16}}{n^{16}} &. 0, 00000000001265907 \\
-\frac{m^{18}}{n^{18}} &. 0, 00000000000070268 \\
-\frac{m^{20}}{n^{20}} &. 0, 00000000000003951 \\
-\frac{m^{22}}{n^{22}} &. 0, 00000000000000224 \\
-\frac{m^{24}}{n^{24}} &. 0, 00000000000000013
\end{aligned}$$

Huius igitur expressionis ope logarithmi sinuum ad duodecim atque etiam plures figuras computari poterunt: si quidem $\frac{m}{n}$ fit $< \frac{1}{2}$ quibus casibus termini duplo pauciores sufficiunt.

Pergamus ergo ad logarithmos cosinuum definiendos, id quod commodissime fiet ex aequatione cos. A. $\frac{m}{n} q = \frac{n^2 - m^2}{n^2} \cdot \frac{5n^2 - m^2}{9n^2} \cdot \frac{25n^2 - m^2}{25n^2} \cdot \text{etc.}$

ex qua fit

$$\begin{aligned}
l \cos. A. \frac{m}{n} 90^\circ &= l \frac{n^2 - m^2}{n^2} \\
&- \frac{m^2}{n^2} \cdot \left(\frac{1}{9} + \frac{1}{27} + \frac{1}{49} + \text{etc.} \right) \\
&- \frac{m^4}{2n^4} \cdot \left(\frac{1}{9^2} + \frac{1}{27^2} + \frac{1}{49^2} + \text{etc.} \right) \\
&- \frac{m^6}{3n^6} \cdot \left(\frac{1}{9^3} + \frac{1}{27^3} + \frac{1}{49^3} + \text{etc.} \right) \\
&- \frac{m^8}{4n^8} \cdot \left(\frac{1}{9^4} + \frac{1}{27^4} + \frac{1}{49^4} + \text{etc.} \right) \\
&\text{etc.}
\end{aligned}$$

feu summis proxime sumendis erit

$$\begin{aligned}
l \cos. A. \frac{m}{n} 90^\circ &= l(n-m) + l(n+m) - 2ln \\
&- \frac{m^2}{n^2} \cdot 0, 23370055013616982735 \\
&- \frac{m^4}{n^4} \cdot 0, 00733901580209602727 \\
&- \frac{m^6}{n^6} \cdot 0, 00048235888031404063 \\
&- \frac{m^8}{n^8} \cdot 0, 00003879475632402982 \\
&- \frac{m^{10}}{n^{10}} \cdot 0, 00000340827260896510
\end{aligned}$$

E e 3

$$\begin{array}{rcl}
 -\frac{m^{12}}{n^{12}} & .0, & 00000031430809718659 \\
 -\frac{m^{14}}{n^{14}} & .0, & 00000002989150274450 \\
 -\frac{m^{16}}{n^{16}} & .0, & 00000000290464467239 \\
 -\frac{m^{18}}{n^{18}} & .0, & 00000000028682639518 \\
 -\frac{m^{20}}{n^{20}} & .0, & 00000000002868076974 \\
 -\frac{m^{22}}{n^{22}} & .0, & 00000000000289697956 \\
 -\frac{m^{24}}{n^{24}} & .0, & 00000000000029506024 \\
 -\frac{m^{26}}{n^{26}} & .0, & 00000000000003026249 \\
 -\frac{m^{28}}{n^{28}} & .0, & 00000000000000312232 \\
 -\frac{m^{30}}{n^{30}} & .0, & 00000000000000032379 \\
 -\frac{m^{32}}{n^{32}} & .0, & 00000000000000003373 \\
 -\frac{m^{34}}{n^{34}} & .0, & 00000000000000000352 \\
 -\frac{m^{36}}{n^{36}} & .0, & 00000000000000000037 \\
 -\frac{m^{38}}{n^{38}} & .0, & 00000000000000000004
 \end{array}$$

Hoc modo igitur reperitur logarithmus hyperbolicus cofinus cuiusque anguli, existente logarithmo finus totius = 0. At logarithmus ordinarius obtinebitur, si iste logarithmus multiplicetur per

$$0,43429448190325182$$

atque ad eum addatur 10. logarithmus scilicet finus totius in tabulis receptus: erit igitur

$$\begin{array}{rcl}
 \log. \text{ cof. } A \cdot \frac{m}{n} 90^\circ & = & 10,0000000000000000 \\
 & - & 2ln + l(n-m) + l(n+m) \\
 -\frac{m^2}{n^2} & .0, & 101494859341892 \\
 -\frac{m^4}{n^4} & .0, & 003187294065451 \\
 -\frac{m^6}{n^6} & .0, & 000209485800017 \\
 -\frac{m^8}{n^8} & .0, & 000016848348597
 \end{array}$$

$$\begin{aligned}
 -\frac{m^{10}}{n^{10}} &. 0, 000001480193986 \\
 -\frac{m^{12}}{n^{12}} &. 0, 000000136502272 \\
 -\frac{m^{14}}{n^{14}} &. 0, 000000012981715 \\
 -\frac{m^{16}}{n^{16}} &. 0, 000000001261471 \\
 -\frac{m^{18}}{n^{18}} &. 0, 000000000124567 \\
 -\frac{m^{20}}{n^{20}} &. 0, 000000000012456 \\
 -\frac{m^{22}}{n^{22}} &. 0, 000000000001258 \\
 -\frac{m^{24}}{n^{24}} &. 0, 000000000000128 \\
 -\frac{m^{26}}{n^{26}} &. 0, 000000000000013 \\
 -\frac{m^{28}}{n^{28}} &. 0, 000000000000001
 \end{aligned}$$

Hinc igitur inuenientur logarithmi vulgares cosinum quorumcunque angulorum, idque ad 14 figuras in fractionibus decimalibus.

Q E I.

§. 29. Ex datis logarithmis sinuum et cosinum inueniuntur primo statim logarithmi secantium et cosecantium. Deinde cum tangentis logarithmus prodeat, si ab aggregato logarithmorum sinus totius et sinus anguli dati subtrahatur logarithmus cosinus, erit pro logarithmis hyperbolicis posito logarithmo sinus totius = 0;

$$l \text{ tang. A. } \frac{m}{n} 90^\circ = l \frac{2n+m}{n+m} + l \frac{2n-m}{n-m} - l \frac{n}{m}$$

$$\begin{aligned}
 &- 0, 934711655830435 \\
 + \frac{m^2}{n^2} &. 0, 072467033424103 \\
 + \frac{m^4}{n^4} &. 0, 004766414748623 \\
 + \frac{m^6}{n^6} &. 0, 000392030432478 \\
 + \frac{m^8}{n^8} &. 0, 000034812963162
 \end{aligned}$$

+

$$\begin{aligned}
& + \frac{m^{10}}{n^{10}} . 0, 000003214019654 \\
& + \frac{m^{12}}{n^{12}} . 0, 000000304294809 \\
& + \frac{m^{14}}{n^{14}} . 0, 000000029357461 \\
& + \frac{m^{16}}{n^{16}} . 0, 000000002875496 \\
& + \frac{m^{18}}{n^{18}} . 0, 000000000285208 \\
& + \frac{m^{20}}{n^{20}} . 0, 000000000028589 \\
& + \frac{m^{22}}{n^{22}} . 0, 000000000002891 \\
& + \frac{m^{24}}{n^{24}} . 0, 000000000000294 \\
& + \frac{m^{26}}{n^{26}} . 0, 000000000000030 \\
& + \frac{m^{28}}{n^{28}} . 0, 000000000000003
\end{aligned}$$

Huius expressionis autem negativum dabit cotangentis anguli $\frac{m}{n} 90^\circ$ logarithmum hyperbolicum. Haecque expressio magnam afferet utilitatem in Hydrographia, in quam ab Halleio logarithmi tangentium sunt introducti.

§. 30 Simili modo logarithmi vulgares tangentium hinc invenientur, erit scilicet

$$\log. \text{ tang. } A. \frac{m}{n} 90^\circ = l \frac{2n+m}{n+m} + l \frac{2n-m}{n-m} - l \frac{n}{m}$$

$$\begin{aligned}
& + 9, 594059885702190 \\
& + \frac{m^2}{n^2} . 0, 031472032735990 \\
& + \frac{m^4}{n^4} . 0, 002070027623789 \\
& + \frac{m^6}{n^6} . 0, 000170256653563 \\
& + \frac{m^8}{n^8} . 0, 000015119077799 \\
& + \frac{m^{10}}{n^{10}} . 0, 000001395831000 \\
& + \frac{m^{12}}{n^{12}} . 0, 000000132153556 \\
& + \frac{m^{14}}{n^{14}} . 0, 000000012749783 \\
& + \frac{m^{16}}{n^{16}} . 0, 000000001248812
\end{aligned}$$

$$\begin{aligned}
 &+ \frac{m^{18}}{n^{18}} \cdot 0,000000000123864 \\
 &+ \frac{m^{20}}{n^{20}} \cdot 0,00000000012416 \\
 &+ \frac{m^{22}}{n^{22}} \cdot 0,0000000001256 \\
 &+ \frac{m^{24}}{n^{24}} \cdot 0,000000000128 \\
 &+ \frac{m^{26}}{n^{26}} \cdot 0,00000000013 \\
 &+ \frac{m^{28}}{n^{28}} \cdot 0,0000000001
 \end{aligned}$$

Quodsi hinc quaeratur logarithmus tangentis anguli 45.
graduum erit $n = 2$ et $m = 1$, fietque summa seriei

$$\begin{aligned}
 &0,0078680081839977 \\
 &1293767264868 \\
 &26602602119 \\
 &590588976 \\
 &13631162 \\
 &322640 \\
 &7782 \\
 &191 \\
 &4
 \end{aligned}$$

$$0,0080001056257721$$

logarithmi vero numerorum naturalium sunt

$$\begin{aligned}
 l_5 &= 0,6989700043360188 \\
 -l_2 &= 0,3010299956639811 \\
 &0,3979400086720377 \\
 \text{addatur } 9,5940598857021902
 \end{aligned}$$

9,9919998943742279
 itemque, 0,0080001056257721

10,0000000000000000

qui est logarithmus tangentis anguli 45. grad.

§. 32. Quodsi quis igitur voluerit tabulas finuum et tangentium eorumque logarithmorum computare ad duodecim figuras in fractionibus decimalibus, dum tabulae vsu receptae eas tantum ad septem figuras exhibent; is sequentibus regulis uti poterit. Propositus scilicet sit angulus $\frac{m}{n}$ 90. graduum erit.

$$\begin{aligned} \text{fin. A. } \frac{m}{n} 90^\circ &= + \frac{m}{n} \cdot 1,5707963267949 \\ &- \frac{m^3}{n^3} \cdot 0,6459640975062 \\ &+ \frac{m^5}{n^5} \cdot 0,0796926262461 \\ &- \frac{m^7}{n^7} \cdot 0,0046817541353 \\ &+ \frac{m^9}{n^9} \cdot 0,0001604411848 \\ &- \frac{m^{11}}{n^{11}} \cdot 0,0000035988432 \\ &+ \frac{m^{13}}{n^{13}} \cdot 0,0000000569217 \\ &- \frac{m^{15}}{n^{15}} \cdot 0,0000000006688 \\ &+ \frac{m^{17}}{n^{17}} \cdot 0,0000000000061 \end{aligned}$$

$$\begin{aligned} \text{col. A. } \frac{m}{n} 90^\circ &= + \frac{m}{n} \cdot 1,00000000000000 \\ &- \frac{m^2}{n^2} \cdot 1,2337005501361 \\ &+ \frac{m^4}{n^4} \cdot 0,2536695079010 \\ &- \frac{m^6}{n^6} \cdot 0,0208634807633 \\ &+ \frac{m^8}{n^8} \cdot 0,0009192602748 \\ &- \frac{m^{10}}{n^{10}} \cdot 0,0000252020424 \end{aligned}$$

+

$$\begin{aligned}
 + \frac{m^{12}}{n^{12}} &. 0, 0000004710875 \\
 - \frac{m^{14}}{n^{14}} &. 0, 0000000063866 \\
 + \frac{m^{16}}{n^{16}} &. 0, 0000000000656 \\
 - \frac{m^{18}}{n^{18}} &. 0, 0000000000005
 \end{aligned}$$

$$\begin{aligned}
 \text{tang. A. } \frac{m}{n} 90^\circ &= \frac{m}{n-m} . 0, 6366197723675 \\
 + \frac{m}{n+m} &. 0, 6366197723675 \\
 + \frac{m}{n} &. 0, 2975567820597 \\
 + \frac{m^3}{n^3} &. 0, 0186886502773 \\
 + \frac{m^5}{n^5} &. 0, 0018424752034 \\
 + \frac{m^7}{n^7} &. 0, 0001975800714 \\
 + \frac{m^9}{n^9} &. 0, 0000216977245 \\
 + \frac{m^{11}}{n^{11}} &. 0, 0000024011370 \\
 + \frac{m^{13}}{n^{13}} &. 0, 0000002664132 \\
 + \frac{m^{15}}{n^{15}} &. 0, 0000000295864 \\
 + \frac{m^{17}}{n^{17}} &. 0, 0000000032867 \\
 + \frac{m^{19}}{n^{19}} &. 0, 0000000003651 \\
 + \frac{m^{21}}{n^{21}} &. 0, 0000000000405 \\
 + \frac{m^{23}}{n^{23}} &. 0, 0000000000045 \\
 + \frac{m^{25}}{n^{25}} &. 0, 0000000000005
 \end{aligned}$$

$$\begin{aligned}
 \text{cot. A. } \frac{m}{n} 90^\circ &= \frac{n}{m} . 0, 6366197723675 \\
 - \frac{n}{2m-n} &. 0, 3183098861837 \\
 - \frac{n}{2n+m} &. 0, 3183098861837 \\
 - \frac{n}{n} &. 0, 2052888894145 \\
 - \frac{m^3}{n^3} &. 0, 0065510747882
 \end{aligned}$$

F f 2

228 METHOD. FACIL. COMPT. ANGVL SINVS

$$\begin{array}{l} - \frac{m^5}{n^5} . 0, 0003450292554 \\ - \frac{m^7}{n^7} . 0, 0000202791060 \\ - \frac{m^9}{n^9} . 0, 0000012366527 \\ - \frac{m^{11}}{n^{11}} . 0, 0000000764959 \\ - \frac{m^{13}}{n^{13}} . 0, 0000000047597 \\ - \frac{m^{15}}{n^{15}} . 0, 0000000002969 \\ - \frac{m^{17}}{n^{17}} . 0, 0000000000185 \\ - \frac{m^{19}}{n^{19}} . 0, 0000000000011 \end{array}$$

$$\log. \sin. A. \frac{m}{n} 90^\circ = l(2n+m) + l(2n-m) + lm - 3ln$$

$$\begin{array}{l} + 9, 5940598857021 \\ - \frac{m^{21}}{n^{21}} . 0, 0700228266059 \\ - \frac{m^{23}}{n^{23}} . 0, 0011172664416 \\ - \frac{m^{25}}{n^{25}} . 0, 0000392291464 \\ - \frac{m^{27}}{n^{27}} . 0, 0000017292708 \\ - \frac{m^{29}}{n^{29}} . 0, 0000000843629 \\ - \frac{m^{31}}{n^{31}} . 0, 0000000043487 \\ - \frac{m^{33}}{n^{33}} . 0, 0000000002319 \\ - \frac{m^{35}}{n^{35}} . 0, 0000000000126 \\ - \frac{m^{37}}{n^{37}} . 0, 0000000000007 \end{array}$$

$$\log. \cos. A. \frac{m}{n} 90^\circ = 10, 0000000050000$$

$$+ l(n+m) + l(n-m) - 2ln$$

$$\begin{aligned}
 -\frac{m^2}{n^2} & . 0, 1014948593419 \\
 -\frac{m^4}{n^4} & . 0, 0031872940654 \\
 -\frac{m^6}{n^6} & . 0, 0002094858000 \\
 -\frac{m^8}{n^8} & . 0, 0000168483486 \\
 -\frac{m^{10}}{n^{10}} & . 0, 0000014801940 \\
 -\frac{m^{12}}{n^{12}} & . 0, 0000001365023 \\
 -\frac{m^{14}}{n^{14}} & . 0, 0000000129817 \\
 -\frac{m^{16}}{n^{16}} & . 0, 0000000012614 \\
 -\frac{m^{18}}{n^{18}} & . 0, 0000000001245 \\
 -\frac{m^{20}}{n^{20}} & . 0, 0000000000126 \\
 -\frac{m^{22}}{n^{22}} & . 0, 0000000000013
 \end{aligned}$$

$$\begin{aligned}
 \log. \text{ tang. } A \frac{m}{n} 90^\circ & = l \frac{2n+m}{n-m} + l \frac{2n-m}{n-m} - l \frac{n}{m} \\
 & + 9, 5940598857022 \\
 + \frac{m^2}{n^2} & . 0, 0314720327359 \\
 + \frac{m^4}{n^4} & . 0, 0020700276238 \\
 + \frac{m^6}{n^6} & . 0, 0001702566535 \\
 + \frac{m^8}{n^8} & . 0, 0000151190778 \\
 + \frac{m^{10}}{n^{10}} & . 0, 0000013958310 \\
 + \frac{m^{12}}{n^{12}} & . 0, 0000001321535 \\
 + \frac{m^{14}}{n^{14}} & . 0, 0000000127498 \\
 + \frac{m^{16}}{n^{16}} & . 0, 0000000012488 \\
 + \frac{m^{18}}{n^{18}} & . 0, 0000000001238 \\
 + \frac{m^{20}}{n^{20}} & . 0, 0000000000124
 \end{aligned}$$

Ff3

+

$$+\frac{m^{22}}{n^{22}}. 0,00000000000012$$

$$+\frac{m^{24}}{n^{24}}. 0,00000000000001$$

Quodsi hic logarithmus a 20. subtrahatur, prodibit logarithmus cotangentis eiusdem anguli $\frac{m}{n} 90^\circ$. Simili autem modo logarithmus cofinus a 20. subtractus relinquet logarithmum secantis, atque logarithmus sinus a 20. subtractus logarithmum cosecantis.